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Lorentz violating effects on a quantized two-level system

M. M. Ferreira Jr.
Universidade Federal do Maranhão (UFMA), Departamento de Física
Campus Universitário do Bacanga, 65085-580, São Luís, Maranhão, Brazil, e-mail: manojr@ufma.br

A.R. Gomes
Departamento de Ciências Exatas, Centro Federal de Educação Tecnológica do Maranhão,
65025-001 São Luís, Maranhão, Brazil, e-mail: argomes@pq.cnpq.br

In this work, we consider the effects of the Lorentz-violating (LV) term $v_\mu\overline{\psi}\gamma^\mu\psi$ belonging to the fermion sector of the extended standard model on the dynamics of a quantum two-level system. We examine how its non-relativistic counterpart, $(\mathbf{p}-e\mathbf{A})\cdot\mathbf{v}/m_e$, affects the Rabi oscillations of a two-level atom coupled with a quantum cavity electromagnetic field. Taking an initial coherent field state in a resonant cavity, it was found that the LV background increases the Rabi frequency and the time interval between collapses and revivals of the population inversion function. It was found that initial field states with low mean number of photons are better probes in order to establish more stringent upper bounds on the background magnitude. In particular, for an initial vacuum state in the cavity the upper limit $\mathrm{v}_x < 10^{-10}eV$ was attained.

I. INTRODUCTION

In the latest years, Lorentz violation (LV) in physical systems has been investigated in connection with a possible breakdown of this symmetry at the Planck scale. Since the demonstration that spontaneous breaking of Lorentz symmetry may occur in the context of string theory [1], small violations of Lorentz covariance in low-energy systems have been searched as a remanent effect of LV at the Planck scale. Naturally, this is a relevant issue, once the Planck scale physics is entirely unknown yet. Nowadays, most LV investigations have been conducted into the framework of the Standard Model Extension (SME) [2], wherein LV is incorporated in all sectors of interaction and governed by tensor coefficients generated as vacuum expectation values of tensor quantities of the original symmetric theory. In such a model, Lorentz breaking takes place only in the particle frame, where such coefficients behave as a set of numbers. In the observer frame, the Lorentz covariance remains valid.

In the framework of the SME, LV has been intensively investigated concerning mainly the photon [3] and fermion sectors with many different purposes, involving radiative corrections [4], topological effects [5], CPT probing experiments [6], hydrogen spectrum [7], and general aspects [8]. Atomic and optical systems [9] (including resonant cavities) have been used as a laboratory to test the limits of Lorentz covariance, implying stringent bounds on the LV coefficients.

The fermion Lagrangian of the SME includes two Lorentz and CPT-odd terms, $v_\mu\overline{\psi}\gamma^\mu\psi, b_\mu\overline{\psi}\gamma_5\gamma^\mu\psi$, with v_μ, b_μ being the LV coefficients generated as vacuum expectation values of tensor quantities belonging to the underlying high energy theory. In a very recent work, it was analyzed the effect of these two terms on a semiclassical two-level system [10], being observed that they yield alterations on its dynamics, inducing sensitive modifications on the usual inversion population function and quantum transitions even in the absence of an external electromagnetic field.

In the present work we consider the effect of the LV term $v_\mu\overline{\psi}\gamma^\mu\psi$ on a quantum atomic system. The starting point is the extended Lagrangian

$$\mathcal{L} = \mathcal{L}_{Dirac} - v_\mu\overline{\psi}\gamma^\mu\psi \tag{1}$$

where $\mathcal{L}_{Dirac}$ is the usual Dirac Lagrangian ($\mathcal{L}_{Dirac} = \frac{1}{2}i\overline{\psi}\gamma^\mu \overleftrightarrow{D}_\mu\psi - m_e\overline{\psi}\psi$). It yields the following nonrelativistic Hamiltonian

$$H = H_{Pauli} + \left[-\frac{(\mathbf{p}-e\mathbf{A})\cdot\mathbf{v}}{m_e}\right]. \tag{2}$$

The purpose here is to examine the effects implied by the Lorentz-violating Hamiltonian in the Rabi nutation of a single atom in the vacuum and in a weak coherent field established in a resonant cavity. Using the amplitude coefficient method, the Schrödinger equation is taken as starting point to obtain differential equations for the amplitude coefficients. These equations govern the dynamics of the two-level system and allow to read the effects induced by the LV background on it. After some approximation, it is derived a system of two coupled typical harmonic oscillator differential equations, whose solution leads to a modified expression for the inversion population function. It then reveals that the Rabi frequency, the collapse (t_c) and revival (t_r) times all increase with the background magnitude. At the same time, the revival packages become larger and more distant from each other. Considering that quantum experiments present a sensitivity of 1 part in 10^{10}, an upper bound $\left(\mathrm{v}_x \leq 10^{-10}eV\right)$ for the background is established.

This paper is outlined as follows. In Sec. II, it is presented a brief resume on the interaction of a two-level system with a quantized monochromatic field. In Sec. III, the Lorentz violating effects stemming from the coupling $v_\mu\overline{\psi}\gamma^\mu\psi$ on the two-level system are properly examined by means of the amplitude coefficient method. Special

attention is paid to the LV effects on the Rabi frequency, collapse and revival times. In Sec. IV, we present our Conclusion and final remarks.

II. INTERACTION OF A TWO-LEVEL ATOM WITH A SINGLE MODE FIELD

First of all, we review a standard result concerning the interaction between a two-level atom with a quantized field (in the absence of the background $\mathbf{v}$). The two levels are identified as $|a\rangle$ and $|b\rangle$ with energy $(1/2)\hbar\omega$ and $-(1/2)\hbar\omega$, respectively. This system is described by the Hamiltonian in the Schrödinger representation (See Ref. [11], Chap. 6):

$$\hat{H} = \hat{H}_{atom} + \hat{H}_{field} + \hat{H}_1, \tag{3}$$

where $\hat{H}_{atom} = \hbar\omega_a|a\rangle + \hbar\omega_b|b\rangle$ is the atomic Hamiltonian, $\hat{H}_{field} = \sum_k \hbar\nu_k(\hat{a}^\dagger\hat{a} + 1/2)$ is the radiation field Hamiltonian, $\hat{H}_1 = -e\mathbf{r}\cdot\mathbf{E}$ describes the atom-field interaction in the dipole approximation, and $\hat{a}, \hat{a}^\dagger$ are the photon destruction and creation operators. Here, $\mathbf{r}$ is the position vector of the electron and $\mathbf{E}$ is the radiation field that interacts with the atom,

$$\mathbf{E} = \sum_k E_{0k}(\hat{a}e^{-i\nu_k t} + \hat{a}^\dagger e^{i\nu_k t})\hat{\epsilon}_k, \tag{4}$$

with $\hat{\epsilon}_k$ being the polarization vector and E_{0k} being the amplitude of the mode of frequency ν_k. Such amplitude is given by $E_{0k} = \sqrt{\hbar\nu_k/(2\epsilon_0\mathcal{V})}$, where $\mathcal{V}$ is the cavity effective volume. This normalization factor is obtained by equating the Fock states energy with the integral over space of the expectation value of the electromagnetic energy density. In this work, we will consider the interaction of a single-mode field of frequency ν with the two-level atom, so from now on we write $\hat{H}_{field} = \hbar\nu(\hat{a}^\dagger\hat{a} + 1/2)$ and $\mathbf{E} = E_0(\hat{a}e^{-i\nu t} + \hat{a}^\dagger e^{i\nu t})\hat{\epsilon}_k$.

Now, the Hamiltonian (3) can be read as $\hat{H} = \hat{H}_0 + \hat{H}_1$, where $\hat{H}_0 = \hat{H}_{atom} + \hat{H}_{field}$ plays the role of the unperturbed interaction and $\hat{H}_1 = -e\mathbf{r}\cdot\mathbf{E}$ can be viewed as a small perturbation. The approach adopted here is developed in the interaction picture, wherein the state vectors evolve with $\hat{H}_1$ whereas the operators evolve with $\hat{H}_0$ [12]. In such a picture, the interaction operator H_1 is to be written as $\hat{H}_{1I}(t) = e^{iH_{0S}t/\hbar}\hat{H}_{1S}e^{-iH_{0S}t/\hbar}$, where $\hat{H}_{1S}$ stands for the atom-field interaction in the Schrödinger picture, in which the operators do not present time dependence (see also Ref. [13], p. 187). For this reason, the time dependence of $\hat{H}_1$ will be dropped out from now on.

In order to evaluate $\hat{H}_1$ in a more suitable form, we use the atom transition operators, $\hat{\sigma}_{ij} = |i\rangle\langle j|$, where $|i\rangle$ represents a complete set of energy eigenstates, so that $1 = \sum |i\rangle\langle i|$. In our two-level case, $|i\rangle = |a\rangle$ or $|b\rangle$, obviously. Considering it, we obtain:

$$\hat{H}_1 = \sum_{ij} g^{ij}\hat{\sigma}_{ij}(\hat{a} + \hat{a}^\dagger), \tag{5}$$

where the electric field was evaluated at $t = 0$ (due to the choice of the Schrödinger representation), $g^{ij} = -e(\mathbf{P}_{ij}\cdot\hat{\epsilon}_k)E_{0k}/\hbar$, and $e\mathbf{P}_{ij} = e\langle i|\mathbf{r}|j\rangle$ is the transition matrix element of the electric dipole moment. Supposing that $\mathbf{P}_{ab} = \mathbf{P}_{ba}, g^{ab} = g^{ba} = g$, the interaction $\hat{H}_1$ takes the form $\hat{H}_1 = g(\hat{\sigma}_{ab} + \hat{\sigma}_{ba})(\hat{a} + \hat{a}^\dagger)$. The terms $\hat{\sigma}_{ab}\hat{a}^\dagger$ and $\hat{\sigma}_{ba}\hat{a}$ should be neglected. Indeed, the term $\hat{\sigma}_{ab}\hat{a}^\dagger$ induces an atomic transition from the ground state ($|b\rangle$) to the excited state ($|a\rangle$) while a photon of frequency ν is emitted. The term $\hat{\sigma}_{ba}\hat{a}$ implies an atomic transition from the excited state ($|a\rangle$) to the ground state ($|b\rangle$) while a photon of frequency ν is absorbed. Both processes do not conserve energy. The exclusion of the non-conserving energy terms is equivalent to the rotating wave approximation (RWA). In the semiclassical theory it takes place a similar fact: the non-resonant terms are neglected. We now introduce the notation: $\hat{\sigma}_+ = \hat{\sigma}_{ab} = |a\rangle\langle b|$, $\hat{\sigma}_- = \hat{\sigma}_{ba} = |b\rangle\langle a|$, so that the energy-conserving Hamiltonian takes the form $\hat{H} = \hat{H}_0 + \hat{H}_1$, with

$$\hat{H}_0 = \hbar\nu\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega\hat{\sigma}_z, \tag{6}$$
$$\hat{H}_1 = \hbar g(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger). \tag{7}$$

The operator $\hat{\sigma}_+$ leads the atom from state $|b\rangle$ to state $|a\rangle$, whereas $\hat{\sigma}_-$ makes the inverse operation. We should also define

$$\hat{\sigma}_z = |a\rangle\langle a| - |b\rangle\langle b|, \hat{\sigma}_x = (\hat{\sigma}_+ + \hat{\sigma}_-), \tag{8}$$
$$\hat{\sigma}_y = -i(\hat{\sigma}_+ - \hat{\sigma}_-), \tag{9}$$

operators which fulfill the Pauli algebra ($[\hat{\sigma}_i, \hat{\sigma}_j] = 2i\epsilon_{ijk}\hat{\sigma}_k$).

Considering the case the electric field is linearly polarized in the x-direction and $\mathbf{P}_{ab}$ is real [14], we can write

$$g = -\frac{eP_{ab}E_0}{\hbar}. \tag{10}$$

The normalization factor is equal to

$$E_0 = \sqrt{\hbar\nu/(2\epsilon_0\mathcal{V})}. \tag{11}$$

Experimentally the cavity set-up provides a precise description for the atomic dynamics even with the atom-field Hamiltonian $\hat{H}_1$ given by Eq. (7), since the interaction with a single mode dominates the evolution.

In the interaction picture, the interaction potential $\hat{V}$ is defined as

$$\hat{V} = \hat{U}_0^\dagger(t)\hat{H}_{1S}\hat{U}_0(t), \tag{12}$$

with $\hat{H}_{1S}$ being the Schrödinger (time independent) representation of $\hat{H}_{1S}$, and $\hat{U}_0(t) = e^{-i\hat{H}_0 t/\hbar}$ is the time evolution operator in this picture. In order to evaluate $\hat{V}$, the Baker-Campbell-Hausdorff lemma is applied and yields:

$$e^{i\nu\hat{a}^\dagger\hat{a}t}\hat{a}e^{-i\nu\hat{a}^\dagger\hat{a}t} = \hat{a}e^{-i\nu t}, \tag{13}$$
$$e^{i\nu\hat{a}^\dagger\hat{a}t}\hat{a}^\dagger e^{-i\nu\hat{a}^\dagger\hat{a}t} = \hat{a}^\dagger e^{i\nu t}, \tag{14}$$
$$e^{\frac{1}{2}i\omega\hat{\sigma}_z t}\hat{\sigma}_\pm e^{-\frac{1}{2}i\omega\hat{\sigma}_z t} = \hat{\sigma}_\pm e^{\pm i\omega t}. \tag{15}$$

Replacing the former relations and Eq. (7) on Eq. (12), we get

$$\hat{V} = \hbar g(\hat{\sigma}_+ \hat{a} e^{i\Delta t} + \hat{a}^\dagger \hat{\sigma}_- e^{-i\Delta t}), \tag{16}$$

with $\Delta = (\omega - \nu)$. In this representation, the wavefunction $|\psi_I(t)\rangle = \widehat{U}_0^\dagger(t)|\psi_S(t)\rangle$ is a linear combination of the two atomic states with arbitrary number of photons (n) in the cavity ($|a,n\rangle$ and $|b,n\rangle$). So we have

$$|\psi_I(t)\rangle = \sum_{n=0}^{\infty}[c_{a,n}|a,n\rangle + c_{b,n}|b,n\rangle]. \tag{17}$$

The Schrödinger equation in the interaction picture $i\hbar|\dot{\psi}_I(t)\rangle = \hat{V}|\psi_I(t)\rangle$ leads to the following system of coupled differential equations:

$$\dot{c}_{a,n} = -ig\sqrt{n+1}e^{i\Delta t}c_{b,n+1}, \tag{18}$$

$$\dot{c}_{b,n+1} = -ig\sqrt{n+1}e^{-i\Delta t}c_{a,n}, \tag{19}$$

which can be easily solved. Considering the atom initially in the excited state $|a\rangle$ and the field with a distribution $c_n(0)$ of photons, we can write $c_{a,n}(0) = c_n(0)$ and $c_{b,n+1}(0) = 0$ to get (see Ref. [11], chap. 6):

$$c_{a,n}(t) = c_n(0)e^{i\Delta t/2}[\cos(\gamma t) - \frac{i\Delta}{\Omega_n}\sin(\gamma t)], \tag{20}$$

$$c_{b,n+1}(t) = -c_n(0)\frac{2ig\sqrt{n+1}}{\Omega_n}\sin(\gamma t)e^{-i\Delta t/2}, \tag{21}$$

where $\Omega_n^2 = \Delta^2 + 4g^2(n+1)$ and $\gamma = \Omega_n/2$. The atomic inversion function can be now defined as

$$W(t) = \sum_{n=0}^{\infty}(|c_{a,n}(t)|^2 - |c_{b,n}(t)|^2), \tag{22}$$

Here, the discrete character of the sum over the number of photons is crucial for the observation of periodic collapses and revivals of the inversion function as a pure quantum effect (see Ref.[15], chap. 10). For the particular case of resonance ($\Delta = 0$) we have

$$W(t) = \sum_{n=0}^{\infty}\rho_{nn}(0)\cos(2\sqrt{n+1}gt), \tag{23}$$

where $\rho_{nn}(0) = |c_n(0)|^2$ is the probability that there are n photons in the cavity at $t = 0$.

Considering an initial coherent state in a cavity with medium number of photons $\bar{n}$ we obtain

$$W(t) = \sum_{n=0}^{\infty}\frac{\bar{n}^n e^{-\bar{n}}}{n!}\cos(2\sqrt{n+1}gt). \tag{24}$$

For $\bar{n} >> 1$ (but not so large, in order to fulfill the RWA condition $g\sqrt{\bar{n}} \ll \omega$ [16]), with small variance Δn, we recover the known Rabi nutation with frequency $\Omega_{\bar{n}}^0 \simeq 2g\sqrt{\bar{n}}$. This is the semiclassical expression as expected from the correspondence principle. For intermediate values of $\bar{n}$, this expression leads to the characteristic behavior of collapses and revivals of the population inversion. This known effect appears mainly due to the interference of the several oscillating patterns associated to different photon numbers.

In particular, at resonance and taking the vacuum ($\rho_{nn}(0) = \delta_{n,0}$) as the initial state in the cavity, we find $W(t) = \cos(2gt)$. So, Rabi oscillations with frequency $\Omega = 2g$ occur due to spontaneous emission, a typical quantum effect that does not occur in the semi-classical system.

III. LORENTZ-VIOLATION EFFECTS

Now we will explore how these quantum fundamental effects can be affected depending on the strength of the Lorentz-breaking coupling in Lagrangian (1). The Hamiltonian can now be written as

$$\hat{H} = \hat{H}_0 + \hat{H}_1 + \hat{H}_1' + \hat{H}_2', \tag{25}$$

where now we have the additional LV contributions: $\hat{H}_1' = e\mathbf{A}\cdot\mathbf{v}/m_e$ and $\hat{H}_2' = -\mathbf{p}\cdot\mathbf{v}/m_e$.

First of all, we analyze the $\hat{H}_1' = e\widehat{A}_x \mathrm{v}_x/m_e$ contribution. After quantization, in the time-independent Schrödinger representation, we can write

$$\hat{H}_1' = \frac{ieE_0\mathrm{v}_x}{m_e\nu}(-\hat{a} + \hat{a}^\dagger). \tag{26}$$

In order to obtain the interaction potential $\hat{V}_1' = \hat{U}_0^\dagger(t)\hat{H}_1'\hat{U}_0(t)$, we use Eqs. (26), (13) and (14), leading to

$$\hat{V}_1' = \frac{ieE_0\mathrm{v}_x}{m_e\nu}(-\hat{a}e^{-i\nu t} + \hat{a}^\dagger e^{i\nu t}). \tag{27}$$

Now we turn to the contribution of $\hat{H}_2' = -\widehat{p}_x \mathrm{v}_x/m_e = -\widehat{\dot{x}}\mathrm{v}_x$. The $\widehat{\dot{x}}$ operator can be rewritten using the Heisenberg equation in the interaction picture as $\widehat{H}_2' = -\mathrm{v}_x/(i\hbar)[\widehat{x}, \widehat{H}_0]$. We found better to represent the $\hat{x}$ operator as

$$\hat{x} = -iP_{ab}\hat{\sigma}_y\hat{\sigma}_z, \tag{28}$$

with $P_{ab} \equiv \langle a|\hat{x}|b\rangle = P_{ab}^* = P_{ba}$, where the operators $\hat{\sigma}_z$, $\hat{\sigma}_y$ are defined in Eqs. (8)-(9). If we represent the energy eigenstates in a vector form

$$|a\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ |b\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{29}$$

we can identify the operators $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ with the Pauli matrices

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{30}$$

Some remarks about the mathematical notation are worthy. Pauli matrix σ_z was already used in Eq. (6) for

the free Hamiltonian as an economic way to describe the atom energy content. Hence, the use of Pauli matrices in this way has no connection with spin operators or spin states. These so-called pseudo-spin operators (Ref. [15], p. 203) are very useful to simplify the calculations and should not lead us to misunderstandings concerning spin magnetic features of the atom states.

In order to achieve a description in the interaction picture, we must apply again the Baker-Campbell-Hausdorff lemma to obtain

$$e^{\frac{1}{2}i\omega\hat{\sigma}_z t}\hat{x}e^{-\frac{1}{2}i\omega\hat{\sigma}_z t} = -iP_{ab}[\sin(\omega t)\hat{\sigma}_x + \cos(\omega t)\hat{\sigma}_y]\hat{\sigma}_z, \tag{31}$$

where Eq. (28) was used. In this representation, the interaction potential

$$\hat{V}_2' = -\frac{\mathrm{v}_x}{i\hbar}[e^{i\hat{H}_0 t/\hbar}\hat{x}e^{-i\hat{H}_0 t/\hbar}\hat{H}_0 - \hat{H}_0 e^{i\hat{H}_0 t/\hbar}\hat{x}e^{-i\hat{H}_0 t/\hbar}], \tag{32}$$

takes the form:

$$\hat{V}_2' = i\mathrm{v}_x P_{ab}\omega[\cos(\omega t)\hat{\sigma}_x - \sin(\omega t)\hat{\sigma}_y]\hat{\sigma}_z. \tag{33}$$

The Schrödinger equation in the interaction picture, $i\hbar|\dot{\psi}_I(t)\rangle = (\hat{V} + \hat{V}_1' + \hat{V}_2')|\psi_I(t)\rangle$, yields a system of coupled differential equations for the probability amplitudes:

$$\begin{aligned}
\dot{c}_{a,n} &= -ig\sqrt{n+1}e^{i\Delta t}c_{b,n+1} \\
&\quad + \frac{eE_0\mathrm{v}_x}{m_e\hbar\nu}(-c_{a,n+1}\sqrt{n+1}e^{-i\nu t} + c_{a,n-1}\sqrt{n}e^{i\nu t}) \\
&\quad - \frac{\mathrm{v}_x P_{ab}}{\hbar}\omega e^{i\omega t}c_{b,n}, \qquad (34)\\
\dot{c}_{b,n+1} &= -ig\sqrt{n+1}e^{-i\Delta t}c_{a,n} \\
&\quad + \frac{eE_0\mathrm{v}_x}{m_e\hbar\nu}(-c_{b,n+2}\sqrt{n+2}e^{-i\nu t} + c_{b,n}\sqrt{n+1}e^{i\nu t}) \\
&\quad + \frac{\mathrm{v}_x P_{ab}}{\hbar}\omega e^{-i\omega t}c_{a,n+1}. \qquad (35)
\end{aligned}$$

with $n = 0, 1, ...\infty$. In general, this system of infinite equations is of difficult solution. Here, we will study the system at resonance ($\Delta = 0$).

For large n we have

$$\begin{aligned}
\dot{c}_{a,n} &= -ig\sqrt{n}c_{b,n} \\
&\quad + \frac{eE_0\mathrm{v}_x}{m_e\hbar\nu}c_{a,n}\sqrt{n}2i\sin(\nu t) \\
&\quad - \frac{\mathrm{v}_x P_{ab}}{\hbar}\omega e^{i\omega t}c_{b,n}, \qquad (36)\\
\dot{c}_{b,n} &= -ig\sqrt{n}c_{a,n} \\
&\quad + \frac{eE_0\mathrm{v}_x}{m_e\hbar\nu}c_{b,n}\sqrt{n}2i\sin(\nu t) \\
&\quad + \frac{\mathrm{v}_x P_{ab}}{\hbar}\omega e^{-i\omega t}c_{a,n}. \qquad (37)
\end{aligned}$$

This must be compared with the differential equations for the coefficients obtained in the semiclassical theory[10]:

$$\dot{a}(t) = i(\Omega_R/2)b(t) + i\alpha_0 a(t)\sin\nu t - \beta_0\omega b(t)e^{i\omega t}, \tag{38}$$

$$\dot{b}(t) = i(\Omega_R/2)a(t) + i\alpha_0 b(t)\sin\nu t + \beta_0\omega a(t)e^{-i\omega t}, \tag{39}$$

where $\alpha_0 = eE_0^{sc}\mathrm{v}_x/(m_e\hbar\nu)$, $\beta_0 = (\mathrm{v}_x P_{ab}/hbar)$, Ω_R is the Rabi frequency and $E_0^{sc} = \sqrt{2n\hbar\nu/\epsilon_0\mathcal{V}}$ is the semiclassical expression corresponding to the normalization factor E_0. Note that $2\sqrt{n}E_0 = E_0^{sc}$ and the correspondence principle is verified.

We consider circular Rydberg atoms in a high Q cavity. In such atoms the long radiative lifetime makes atomic relaxation negligible during the atom's transit time across the cavity [17]. Also a high Q cavity turns the photon lifetime longer than the atom-cavity interaction time. Brune et al. [18] investigated the resonant effects of the vacuum in a cavity mode. There the authors considered the transition frequency $\omega = 51GHz$ for $n = 50$ circular Rydberg atoms. The matrix element between the circular levels 50 and 51 of a linear projection of the electric dipole on the orbit's plane has the large value $eP_{ab} = 1250ea_0$, where a_0 is the Bohr radius. This means a very large classical radius and a very large radiative decay time. The effective cavity volume was $0.7cm^3$. With it, Eq. (11) provides a vacuum field amplitude at antinodes of $E_0 = 6.95 \times 10^{-4}V/m$. These parameters correspond to the following vacuum Rabi oscillation frequency $\Omega_{vac} = 2g = 2eP_{ab}E_0/\hbar = 132kHz$, which implies $g = 66kHz$ for the coupling constant. In these estimates we are neglecting the usual spatial variation of the electromagnetic field in the cavity. We will also consider that the applied field frequency is near resonance ($\nu = \omega = 51GHz$).

Now, we can use the above values for the parameters to estimate the relative importance of the Lorentz-violating terms in Eqs. (34)-(35). In this way, we have $eE_0/(m_e\hbar\omega) = 2 \times 10^{31}kg^{-1}m^{-1}$ and $P_{ab}\omega/\hbar = 3 \times 10^{37}kg^{-1}m^{-1}$. Then, we note that for $\bar{n} \ll 10^{12}$ the magnitude of the second term (corresponding to the influence of $-\mathbf{p}\cdot\mathbf{v}/m_e$) can be taken as much larger than the magnitude of the first one (corresponding to $e\mathbf{A}\cdot\mathbf{v}/m_e$). At resonance, both terms oscillate with the same frequency, but the smaller amplitude argument is an enough reason to neglect the term stemming from $e\mathbf{A}\cdot\mathbf{v}/m_e$. The fact that such term does not imply quantum effects at a first approximation is in agreement with the semi-classical behavior [10] associated with it. Indeed, it amounts only at phase effects without altering the semi-classical population inversion function. Neglecting this term, it is ascribed an explicit gauge-independent character for the LV modifications.

From these considerations the equations for the probability amplitudes can be properly approximated as

$$\begin{aligned}
\dot{c}_{a,n} &\simeq -ig\sqrt{n+1}c_{b,n+1} \\
&\quad - \frac{\mathrm{v}_x P_{ab}}{\hbar}\omega e^{i\omega t}c_{b,n}, \qquad (40)\\
\dot{c}_{b,n+1} &\simeq -ig\sqrt{n+1}c_{a,n} \\
&\quad + \frac{\mathrm{v}_x P_{ab}}{\hbar}\omega e^{-i\omega t}c_{a,n+1}. \qquad (41)
\end{aligned}$$

Here, an interesting issue is to know if the high fre-

quency of the Lorentz-violating term can in fact provide an effective correction. In fact, we remind that a high-frequency term proportional to 2ω was discarded due to the rotating-wave approximation. The exclusion of the counter-rotating terms from the equations of motion is justified when the frequency of the external modes most strongly interacting with the system is very large compared to the strength g of the interaction [15] (in the present case $\omega = 51GHz$ whereas $g = 66kHz$). We have already seen that such exclusion is equivalent to neglecting the non-energy-conserving processes such as the excitation of an atom along with the emission of a photon [15]. From this point of view the Lorentz-violating term induced by the background has an energy non-conserving character, once the Hamiltonian of Eq. (26) amounts at creation and annihilation of a photon without a corresponding change on the atomic level. One possibility is to consider physical situations where such an energy-violating terms grows in importance (see [19], p. 151) and [20]. However, as the terms depending on the background oscillate with frequency ω whereas the discarded terms from the RWA oscillate with 2ω, the investigation of the influence of the background on the modified equations of motion (Eqs. (40)-(41)) seems to be a sensible option.

We can decouple those equations using the approximation that during the time interval of measurement of Rabi nutation the oscillating terms are averaged to zero $[\langle\cos(\omega t)\rangle = \langle\sin(\omega t)\rangle = 0]$. This gives

$$\ddot{c}_{a,n} \simeq \left(-g^2(n+1) - \frac{\mathrm{v}_x^2 P_{ab}^2 \omega^2}{\hbar^2}\right) c_{a,n}\,, \quad (42)$$

$$\ddot{c}_{b,n+1} \simeq \left(-g^2(n+1) - \frac{\mathrm{v}_x^2 P_{ab}^2 \omega^2}{\hbar^2}\right) c_{b,n+1}\,. \quad (43)$$

We consider as the initial state the atom in the excited state with the cavity with a field characterized by coefficients $c_n(0)$, so that the results are

$$c_{a,n} \simeq c_{a,n}(0)\cos\left[\zeta_n\sqrt{n+1}gt\right], \quad (44)$$

$$c_{b,n+1} \simeq -ic_{a,n}(0)\sin\left[\zeta_n\sqrt{n+1}gt\right]. \quad (45)$$

with

$$\zeta_n \equiv \sqrt{1 + \frac{1}{n+1}\left(\frac{\mathrm{v}_x P_{ab}\omega}{\hbar g}\right)^2} \quad (46)$$

The average number of photons is

$$\bar{n}(t) = \sum_0^N (nP_n(t)).$$

where $P_n(t) = |c_{a,n}|^2 + |c_{b,n}|^2$ is the probability for finding n photons in the cavity. Now note from Eqs. (44)-(45) that $P_n(t) = P_n(0)$. This shows that the photon statistics and consequently the average number of photons is not altered by the background v_x.

Considering an initial vacuum state in a cavity at resonance ($\Delta = 0$), we attain the following population inversion function:

$$W(t) = \cos(2g\zeta_0 t). \quad (47)$$

For a sufficiently small background ($\mathrm{v}_x P_{ab}\nu/\hbar g \ll 1$), it is allowed to write

$$\zeta_0 \cong 1 + \frac{1}{2}\frac{\mathrm{v}_x^2 P_{ab}^2 \nu^2}{\hbar^2 g^2} \quad (48)$$

The former Eqs.(47)-(48) mean that, at first approximation, there appears an effective coupling (due to the background) given by $g' = g + \mathrm{v}_x^2 P_{ab}^2 \nu^2/(2\hbar^2 g)$. Consequently, this implies an increasing on the value of Rabi frequency:

$$\Omega_0 = 2g\left(1 + \frac{1}{2}\frac{\mathrm{v}_x^2 P_{ab}^2 \nu^2}{\hbar^2 g^2}\right) \quad (49)$$

Regarding that discrepancies of 1 part in 10^{10} from the usual results of quantum mechanics can be detected, we can limit the LV effects in accordance with this sensitivity. We then impose that the correction term should be smaller than 10^{-10}, that is $(\mathrm{v}_x P_{ab}\nu/\hbar g) < 10^{-10}$. For the chosen parameters $P_{ab}\omega/\hbar = 3.2 \times 10^{37} kg^{-1}m^{-1}$ and $g = 66kHz$, **s**uch condition leads to the following upper bound on the LV background: $\mathbf{v}_x < 2.06 \times 10^{-38} kgm/s$, or in natural units $\mathrm{v}_x < 10^{-10}eV$**.**

The same effect can be studied for an initial coherent state in a cavity with medium number of photons $\bar{n}$ and at resonance ($\Delta = 0$). In this case we attain the following population inversion function:

$$W(t) = \sum_{n=0}^{\infty}\frac{\bar{n}^n e^{-\bar{n}}}{n!}\cos\left[2\sqrt{n+1}g\sqrt{1 + \frac{\mathrm{v}_x^2 P_{ab}^2 \omega^2}{(n+1)\hbar^2 g^2}}t\right]. \quad (50)$$

This expression allows to infer that the net effect of the background on the probability amplitudes is the increasing of the frequency of the collapses and revivals of the population inversion. It is instructive to note how the high frequency terms of opposite phases in Eqs. (40)-(41) conspired to provide such a correction. Note also that for $\mathrm{v}_x = 0$ the usual result for the inversion $W(t)$ - a superposition of frequencies $2\sqrt{n+1}g$ - is recovered for an initial coherent state (see Eq. (23)). For large $\bar{n}$, the sum in Eq. (50) can be simplified to produce $W(t) \sim \cos(\Omega_{\bar{n}} t)$, where

$$\Omega_{\bar{n}} = 2\sqrt{\bar{n}}g\sqrt{1 + \frac{\mathrm{v}_x^2 P_{ab}^2 \omega^2}{\bar{n}\hbar^2 g^2}} \quad (51)$$

is the Rabi frequency corrected by the Lorentz-violating background. This expression reveals that the Rabi frequency increases with the background magnitude. For small background we can also write

$$\Omega_{\bar{n}} \cong 2\sqrt{\bar{n}}g\left(1 + \frac{1}{2}\frac{\mathrm{v}_x^2 P_{ab}^2 \omega^2}{\bar{n}\hbar^2 g^2}\right) \quad (52)$$

An important point here is the appearance of the mean photon number factor ($\bar{n}$) in the correction term in comparison with the previous case of an initial vacuum state in the cavity (see Eq. (49)). Indeed, the larger the average number of photons for an initial coherent state in the cavity, the lower is the correction the Rabi frequency induced by the fixed background. Considering the same experimental sensitivity of 1 part in 10^{10} in a measurement of the Rabi frequency, a larger bound (less stringent) on v_x may now be achieved. In fact, for a state with large number of photons ($\bar{n} >> 1$), the allowed background upper value is multiplied by $\sqrt{\bar{n}}$ ($\mathrm{v}_x^{COH} = \sqrt{\bar{n}}\mathrm{v}_x^{VAC}$). This means that cavity experiments with smaller number of photons imply better bounds (more stringent) on the background magnitude. In this sense, the best probes for determining LV deviations from usual quantum mechanics are really the vacuum states.

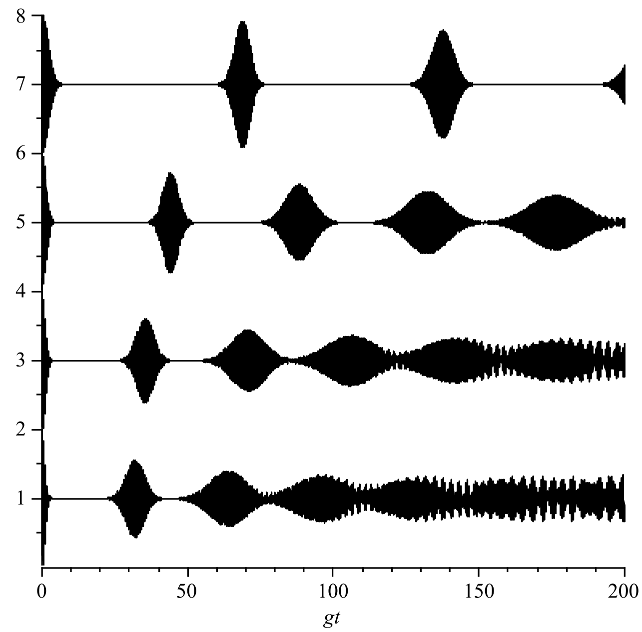

FIG. 1: Population inversion as a function of time $C+W(t)$, for $\bar{n}=25$ and (a) $C=1, \nu_x=0$ (first, lower curve), (b) $C=3, \nu_x=5\times10^{-33}kgm/s$ ($2.5\times10^{-5}eV$ in natural units - second curve), (c) $C=5, \nu_x=1\times10^{-32}kgm/s$ ($5\times10^{-5}eV$ in natural units - third curve) and (d) $C=7, \nu_x=2\times10^{-32}kgm/s$ (or $\nu_x=1\times10^{-4}eV$ - fourth, upper curve). The C constants are included to make easier the comparison.

To study the intermediate scale where effects of collapses and revivals appear and to see the influence of the extra factor depending on the background v_x on the expression of Eq. (50), we proceed with a graphical analysis. In Fig. 1, we present a plot of $W(t)\times gt$. Such a figure shows a sequence of four curves for an initial coherent state with $\bar{n}=25$ and coupling $g=66kHz$. The first lower curve shows the usual sequence of collapses and revivals in the absence of Lorentz violation ($\mathrm{v}_x=0$). As the background increases, the collapses and revivals tend to occur later. Note also that the sequence of collapses and revivals is always destroyed after some time, larger for higher values of the background. These observations indicate that a stronger background favours the maintenance of the revival/collapse sequence for a greater time.

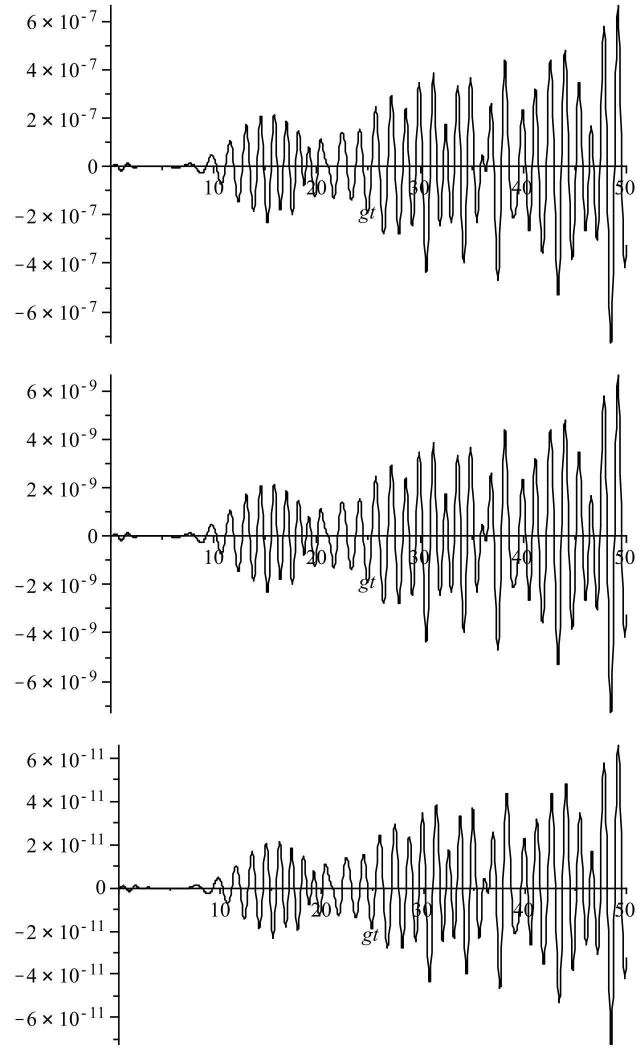

FIG. 2: Difference $W-W_0$ between population inversion functions W for v_x and W_0 for $\mathrm{v}_x=0$, for $\bar{n}=25$ where (a) $\nu_x=5\times10^{-37}kgm/s$ ($2.5\times10^{-9}eV$ in natural units - upper curve), (b) $\nu_x=5\times10^{-38}kgm/s$ ($2.5\times10^{-10}eV$ in natural units - middle curve), (c) $\nu_x=5\times10^{-39}kgm/s$ ($2.5\times10^{-11}eV$ in natural units - lower curve)

An interesting issue is to verify if this kind of analysis is able to impose an upper bound on the background magnitude, in agreement to the analytical result of Eq. (52). A reasonable criterion consists in taking the maximum background value that does not yield significative discrepancy (of 1 part in 10^{10}) on the population inversion pattern taking as reference the usual case ($\mathrm{v}_x=0$). In this way, we have chosen to plot in Fig. 2 the difference $W-W_0$ at the time scale $0<gt<50$, sufficient to

reveal the beginning of the second collapse in the usual case (see lower picture in Fig. 1). The comparative results from Fig. 2 shows that $\mathrm{v}_x < 10^{-10} eV$ is an efficient condition in keeping the difference $W - W_0$ below 10^{-10} for the first observed sequence of collapses and revivals of the population inversion. This is in agreement with our previous estimate for the bound for an initial vacuum state in the cavity.

Fig. 3 shows the same kind of graphical analysis of Fig. 1 for an initial coherent state with lower mean number of photons ($\bar{n} = 5$). Here, the background effect of keeping the sequence of collapses and revivals for longer times is more clearly depicted. Further, it is seen that the collapse time and separation time (between the revivals) increase with the background magnitude. However, even lower background values already reveal larger separations between collapses and revivals (in comparison with the case of Fig. 1).

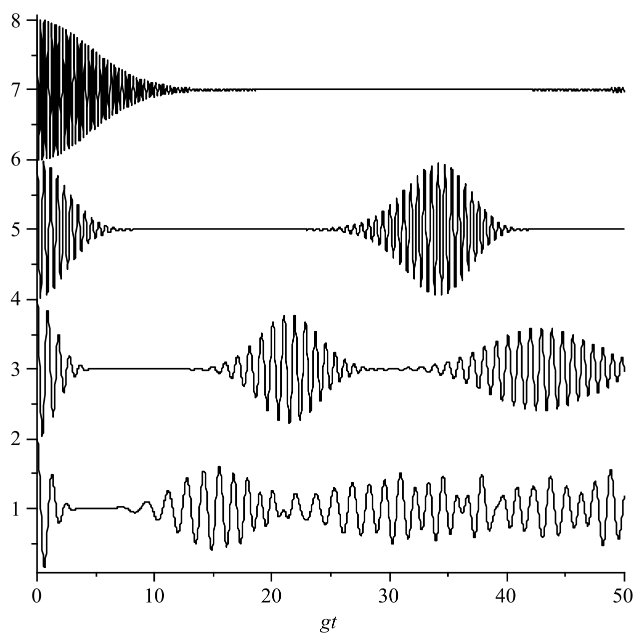

FIG. 3: Population inversion as a function of time $C + W(t)$, for $\bar{n} = 5$. Values for C and v_x are the same from Fig. 1.

A further step is to use Eq. (50) to study the implied changes on the known expressions for the times t_R (period of Rabi oscillations), t_c (collapse time) and t_r (revival time), defined in Ref. [11]. As before, we consider $\bar{n} \gg 1$. To measure the influence of the background, it will be used the dimensionless parameter $\alpha = \mathrm{v}_x P_{ab} \omega / (\hbar g)$. The time period t_R is given by the inverse of Rabi frequency

$$t_R \sim \frac{1}{\Omega_{\bar{n}}} = \frac{1}{\Omega^0_{\bar{n}}} \left(1 - \frac{\alpha^2}{\bar{n}} \right), \qquad (53)$$

outcome obtained using Eq. (52), with $\Omega^0_{\bar{n}} = 2g\sqrt{\bar{n}}$. It shows that t_R is reduced by the presence of the background. This effect can be seen verifying that all the upper curves of Fig. 3 exhibit higher oscillation frequency in comparison with the lower ones (corresponding to lower v_x values).

Another parameter is the time of collapse of oscillations (t_c), that is, the time in which the oscillations associated with different values of n become uncorrelated. For an initial coherent state in the cavity with sufficiently large $\bar{n}$ (for photon number standard deviation $\Delta n = \sqrt{\bar{n}} \ll \bar{n}$), we can estimate t_c as (see Ref. [11]): $(\Omega_{\bar{n}+\sqrt{\bar{n}}} - \Omega_{\bar{n}-\sqrt{\bar{n}}}) t_c \sim 1$. After using Eq. (52), we write

$$t_c \sim t^0_c \left(1 + \frac{\alpha^2}{2\bar{n}} \right), \qquad (54)$$

where $t^0_c = 1/(2g)$ is the collapse time in the usual two-level quantum system (without LV). This expression shows an enlargement of the collapse time with the background magnitude with fixed $\bar{n}$. Indeed, the upper curves of Fig. 1 show longer collapse times when compared with the lower ones. Also, when comparing Figs. 1 and 3 for the same background values, we verify that the collapse time decreases with an increasing value of $\bar{n}$. This is consistent with Eq. (54). This effect is similar to the collapse time behavior observed in the usual two-level quantum system (without LV) at a non-resonant regime $(\Delta \neq 0)$, which also diminishes with $\bar{n}$ (see ref. [11]).

Finally, we regard the time of revival of oscillations, (t_r), given by the condition $(\Omega_{\bar{n}} - \Omega_{\bar{n}-1}) t_r = 2\pi m,\ m = 1, 2, ...,$ as the time in which two oscillators with neighboring photon numbers $n = \bar{n} - 1$ and $n = \bar{n}$ acquire a $2m\pi$ phase difference [21]. This gives

$$t_r \sim t^0_r \left(1 + \frac{\alpha^2}{2\bar{n}} \right) \qquad (55)$$

with $t^0_r = 2\pi m \sqrt{\bar{n}}/g,\ m = 1, 2, 3, ...$ This result turns clear that for a fixed background the revivals take place at regular intervals as in the usual two-level quantum theory. Also, such intervals are augmented for increasing values of the background, becoming the revival packages more distant (in time) from each other. This is compatible with the behavior exhibited by the upper curves in Figs. 1 and 3.

IV. CONCLUSION

In this work, we have considered the main consequences of the LV vector coupling term $\left(v_\mu \overline{\psi} \gamma^\mu \psi \right)$ on a quantized two-level atom coupled with a quantized electromagnetic field in a cavity. We have written the nonrelativistic LV corrections in the interaction picture and considered such contributions into the Schrödinger equation in order to obtain the modified system of coupled differential equations for the probability amplitude coefficients that describe the atom-field wave function. Experimental values of the relevant parameters of the model

revealed that the term $\mathbf{A}\cdot\mathbf{v}$ is of lower magnitude when compared with the term $\mathbf{p}\cdot\mathbf{v}$, which corroborates recent results from the semiclassical theory. The Lorentz-violating Hamiltonian stemming from the quantized vector potential has a "non-conserving" energy character. This kind of non-conserving terms are usually discarded in the rotating wave approximation, but here are the ones that implied physical effects on the system.

After decoupling the system of differential equations, the probability amplitude coefficients fulfill typical harmonic oscillator equations, whose solutions lead to modified expressions for the population inversion function and for the Rabi frequency. These results allow to note that the photon statistics and the average number of photons in the cavity are not changed. On the other hand, the Rabi frequency increases with the background value, which is associated with a decreasing in the Rabi period (t_R). At the same time, the modified population inversion function revealed that the revivals tend to occur later as the background magnitude increases. As a consequence of the alteration implied on the Rabi frequency, the collapse (t_c) and revival (t_r) times increase with an increasing background, so that the revival packages become larger and more distant from each other. In order to keep these modifications in an undetectable scale for the parameter ranges considered, an upper bound $\left(\mathrm{v}_x \leq 10^{-10}eV\right)$ for the background was set up. Such bound is in agreement with a recent result obtained by us from a semiclassical approach [10].

Note also that the deduced expressions for the modified times t_R, t_c and t_r are better verified for $\bar{n} >> \sqrt{\bar{n}}$. In our example, we have chosen $\bar{n} = 25$ and $\bar{n} = 5$, which does not fulfill this condition. However, these not so large $\bar{n}$ values have already obeyed qualitatively the tendency stated by Eqs. (53), (54) and (55). Despite a choice of larger values of $\bar{n}$ has implied a longer sequence of collapses and revivals, our analysis showed that smaller values of $\bar{n}$ are more efficient in the task of establishing a more stringent upper bound on the background magnitude.

Some interesting issues may be regarded in forthcoming investigations, as the examination of the spontaneous emission in the presence of Lorentz violation, with the evaluation of LV corrections on the decaying rate.

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